Lecture 3 - Motion at constant acceleration
- an important special case

An example of motion at constant acceleration is motion near the earth's surface due to gravity. To restrict this motion to one dimension we consider just up and down motion. Later when we study motion in two dimensions, we will analyse projectile motion (eg shoot the monkey).

It is possible to write down formulas for displacement and velocity when the acceleration is constant. To do this, first notice that if the acceleration, a, is constant, then the velocity increases at a constant rate. This means that if we plot velocity as a function of time it is a straight line with slope a. We thus deduce that the velocity as a function of time has the form,

$$v = v_0 + at \tag{1}$$

Now the average velocity over a time period, t, is given by,

$$\overline{v} = \frac{1}{2}(v_0 + v) = \frac{\Delta x}{t} \tag{2}$$

Here we used Eq. (2) of Lecture 2, but used t instead of Δt. We also dropped the subscipts used in Lecture 2. This makes the formula more compact, but we have to keep in mind the meaning of each of the variables. Using Eq. (1) for v in Eq. (2), we find,

$$\frac{1}{2}(v_0 + v_0 + at) = \frac{\Delta x}{t}, \tag{3}$$

and solving for Δx, leads to,

$$\Delta x = v_0 t + \frac{1}{2}at^2. \tag{4}$$

We may also eliminate t from this equation in favor of v using Eq. (1), after some algebra this yields,

$$v^2 = v_0^2 + 2a\Delta x. \tag{5}$$

Eqs. (1), (2), (4) and (5) are the key constant acceleration formula and they appear in many problems due to their broad importance. These formula will also be the basis of understanding many types of motion in two dimensions.

An application - Whooping it up

An excited cowboy fires his handgun(at head height) vertically upward with an initial speed $v_0 = 311m/s$. Ignoring air drag on the bullet, how long before the bullet lands on the cowboy's head? How high does the bullet go? Take the gravitational acceleration to be $-9.81m/s^2$. (Note that this acceleration is the same regardless of whether the motion is positive (up) or negative (down)) To find the time taken for the bullet to reach its highest altitude note that at the highest point of its motion, $v = 0$ so we must have(using Eq. (1)),

$$0 = 311m/s - 9.81m/s^2t \tag{6}$$

which gives $t = 31.7s$. The time until the bullet hits the cowboy's head is then $2t = 63.4s$. The displacement at the top of its trajectory is found from Eq. (4), which yields $\Delta x = 4930m$ (about 3 miles!). What is the velocity of the bullet when it hits the cowboy? Answer $311m/s$! How would these results be affected by air drag?